MINISTRY OF EDUCATION AND TRAINING

HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY AND EDUCATION

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DEVELOPMENT OF THE THIN-WALLED COMPOSITE BEAM MODEL UNDER MECHANICAL AND THERMAL LOADS

PH.D THESIS SUMMARY

MAJOR: ENGINEERING MECHANICS

HO CHI MINH CITY, APRIL 2025

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I acknowledge the support I have received for my research through the guidance of Prof. Dr. Trung-Kien Nguyen and Dr. Do Tien Tho.

Xuan-Bach Bui

List of publications

ISI papers with peer-reviews:

1. **Bui, X.-B.**, T.-K. Nguyen, N.-D. Nguyen, and T.P. Vo, *A general higher-order shear deformation theory for buckling and free vibration analysis of laminated thin-walled composite I-beams.* Composite Structures, 2022. 295: p. 115775. https://doi.org/10.1016/j.compstruct.2022.115775

2. **Bui, X.-B.**, T.-K. Nguyen, and P.T.T. Nguyen, *Stochastic vibration and buckling analysis of functionally graded sandwich thin-walled beams.* Mechanics Based Design of Structures and Machines, 2023: p. 1-23. https://doi.org/10.1080/15397734.2023.2165101

3. **Bui, X.-B.**, T.-K. Nguyen, A. Karamanli, and T.P. Vo, *Size-dependent behaviours of functionally graded sandwich thin-walled beams based on the modified couple stress theory.* Aerospace Science and Technology, 2023. 142: p. 108664. https://doi.org/10.1016/j.ast.2023.108664

4. **Bui, X.-B**. and T.-K. Nguyen, *Deterministic and stochastic flexural behaviors of laminated composite thin-walled I-beams using a sinusoidal higher-order shear deformation theory.* Mechanics Based Design of Structures and Machines, 2023: p. 1-30. https://doi.org/10.1080/15397734.2023.2297840

5. **Bui, X.-B.**, P.T.T. Nguyen, and T.-K. Nguyen, *Spectral projection and linear regression approaches for stochastic flexural and vibration*

analysis of laminated composite beams. Archive of Applied Mechanics, 2024 .https://doi.org/10.1007/s00419-024-02565-x

Domestic papers with peer reviews and conference papers:

6. **Bui, X.B**., T.K. Nguyen, Q.C. Le, and T.T.P. Nguyen. *A novel two-variable model for bending analysis of laminated composite beams*. in *2020 5th International Conference on Green Technology and Sustainable Development (GTSD)*. 2020.

7. **Bui, X.-B.,** A.-C. Nguyen, N.-D. Nguyen, T.-T. Do, and T.-K. Nguyen, *Buckling analysis of laminated composite thin-walled I-beam under mechanical and thermal loads.* Vietnam Journal of Mechanics, 2023. 45(1): p. 75-90. https://doi.org/10.15625/0866-7136/17956

8. **Bui, X.-B**., T.-K. Nguyen, T.T.-P. Nguyen, and V.-T. Nguyen. *Stochastic Vibration Responses of Laminated Composite Beams Based on a Quasi-3D Theory*. in *ICSCEA 2021*. 2023. Singapore: Springer Nature Singapore.

Nomenclature

b, h, L: Width, height, length of the rectangular solid beam

u, w: axial and transverse displacements at any point on the rectangular solid beam

$\bar{u}, \bar{w}$: axial and transverse displacements at mid-plane of rectangular solid beams

$\mathbf{K}$: stiffness matrix

$\mathbf{M}$: mass matrix

$\mathbf{f}$: external force vector

b_1, b_2, b_3: the widths of the upper flange, web, and lower flange respectively of the I- or channel thin-walled beams

h_1, h_2, h_3: the thicknesses of the upper flange, web, and lower flange respectively of the I- or channel thin-walled beams

ϕ : rotational angle about the pole axis

E_1, E_2: Young's moduli

G_{12}, G_{23}, G_{13}: shear moduli

$\nu, \nu_{12}, \nu_{13}, \nu_{23}$: Poisson's ratios

ρ : mass density

$F_\omega(s)$: the warping function of thin-walled beams

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Abstract

Thin-walled beams are widely used in engineering fields like civil, aerospace, and automotive for their load capacity and lightness. This thesis investigates their structural responses, focusing on cross-section shapes, static analysis (deflection, buckling stability under thermal and mechanical loads), and vibration analysis (fundamental frequencies and mode shapes, particularly torsional modes for open-sections). It aims to enhance design, optimization, and safety in using advanced composite materials by predicting beam responses to various loads, material uncertainties, shear strain, and size effects. Previous models like Vlasov's and first-order shear deformable beam theories are extended by proposing a high-order theory for composite beams. This model supports stochastic analysis (considering material property variations) and size-dependent effects analysis (using modified couple stress theory for microbeams). Techniques include a new beam solver, polynomial chaos expansion, and artificial neural networks for efficient and accurate response evaluation. Sensitivity analysis evaluates material property uncertainties' impact. The findings offer benchmarks for future research. Validation precedes these analyses, and MATLAB is used for all computations, prioritizing accuracy and efficiency.

1. Scope

1.1 Composite material

Composite materials have emerged as a core element in modern engineering and materials science, revolutionizing the way we design and manufacture a diverse range of structures and products. Unlike homogeneous materials, composites are fabricated by combining two or more distinct materials, each contributing its unique properties to create a synergistic material with enhanced characteristics. This blending of materials enables the development of materials that surpass the limitations of individual constituents, offering a remarkable balance of strength, stiffness, and versatility.

The state-of-the-art manufacturing techniques enable engineers to fabricate many kinds of composites. In the later sections, functionally graded composite (FGC), laminated composite (LC), and the porous metal foam are deeply analysed and discussed. These composites find applications in numerous fields, including aerospace, automotive, sport equipments, and structural engineering. They are particularly beneficial in components exposed to extreme conditions or varying loads, where a uniform material may not provide optimal performance.

1.2 Thin-walled beams

Thin-walled beams are structural elements characterized by having a relatively small ratio of wall thickness to their other dimensions, such as length and width, distinguising them with solid or thick-walled counterparts. The use and design of thin-walled beams is always driven by the need for structural efficiency, as the minimal use of material helps reduce weight while maintaining adequate strength and stiffness. For centuries, steel thin-walled beams have been used for building and bridges structures. Their behaviours and design are very well-studied as steel buildings constantly reach new heights and bridges keep increasing their span length. Nonetheless, when the newly introduced composite material are applied into thin-walled structures and the demand for structural efficiency grows, the research for composite thin-walled structures have a lot more gaps to fill. This study aims to analyse these composite thin-walled beam sections under mechanical and thermal loads.

1.3 Uncertainty quantification

In real-world scenarios, fluctuations in component materials due to production processes or unforeseen elements necessitate accounting for uncertainty to enhance beam response prediction reliability. Uncertainty quantification (UQ) addresses variability and imprecision in engineering models. Three approaches to UQ are utilized: Monte Carlo Simulation (MCS), Polynomial Chaos Expansion (PCE), and Artificial Neural Network (ANN).

MCS involves running numerous simulations with randomly generated input parameters, providing a distribution of possible beam response outcomes. While accurate, MCS can be computationally intensive. PCE and ANN offer more efficient alternatives, requiring fewer simulations to capture uncertainties and provide accurate predictions.

In addition to uncertainty quantification, sensitivity analysis examines the impact of each input parameter and their interactions on beam responses. Comparisons between MCS, PCE, and ANN are made based on the Sobol indices of beam simulations. Further details on these comparisons are discussed in subsequent chapters.

2. Theory overview

2.1 Thin-walled beam theory

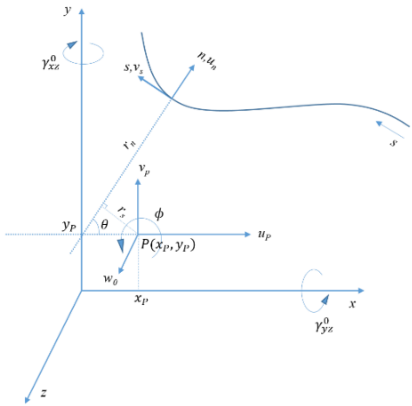

Figure: Thin-walled coordinate systems

Based on the definition of Vlasov [10], thin-walled beams are beams with $\frac{h}{l} \leq 0.1$ and $\frac{l}{L} \leq 0.1$, where h is the wall thickness, l is any characteristic dimension of the cross-section, and L is the beam length. The wall thickness can only vary along the beam's cross section contour, but remains constant along the beam span. A same set of coordinates for the analysis of thin-walled beams is used throughout this thesis. Cartesian coordinate system (x, y, z), local plate coordinate system (n, s, z) and contour coordinate s along the profile of the section are considered. It is assumed that θ is an angle of orientation between (n,s,z) and (x,y,z) coordinate systems, the pole P with coordinates (x_P, y_P) is the shear center of the section.

-Classical thin-walled beam theory (CTWBT): is one of the earliest approaches to analyzing thin-walled structures. It is based on the fundamental assumptions of Euler-Bernoulli beam theory, where plane sections remain plane and perpendicular to the neutral axis after deformation. In CTWBT, the influence of shear deformation is neglected, and the beam is primarily analyzed under bending and torsion using simplified warping assumptions. While this theory provides a good approximation for slender beams with thin walls, it becomes less accurate for cases involving significant shear deformation or warping effects, particularly in non-uniform or composite structures.

-First-order thin-walled beam theory (FTWBT): To improve upon the limitations of CTWBT, the First-Order Thin-Walled Beam Theory (FTWBT) incorporates shear deformation effects using the Timoshenko beam assumption. This theory accounts for transverse shear flexibility, making it more suitable for beams with moderate thickness and non-uniform cross-sections. FTWBT introduces a more refined treatment of warping deformation, allowing for a better prediction of stresses and displacements in beams subjected to bending, torsion, and axial loads. However, it still assumes a linear variation of shear strain across the beam thickness, which may not be sufficient for highly heterogeneous materials or complex loading conditions.

-High-order thin-walled beam theory (HTWBT): further refines the analysis by incorporating higher-order displacement fields and shear deformation effects. Unlike FTWBT, which assumes a linear shear strain distribution, HTWBT allows for nonlinear warping and a more accurate representation of through-thickness strain variation. This approach is particularly beneficial for composite and functionally graded materials, where stiffness variations across the thickness play a crucial role in structural behavior.

The progression from CTWBT to HTWBT reflects the increasing need for accuracy in modeling thin-walled structures, especially in modern engineering applications involving composites, heterogeneous materials, and micro-scale structures. While CTWBT provides a fundamental understanding, FTWBT introduces necessary shear corrections, and HTWBT offers a more sophisticated approach for capturing complex deformations. Selecting the appropriate theory depends on the structural configuration, material properties, and the required level of precision for a given application.

2.2 Materials

There are three main types of composote materials used in this thesis: laminated composite material, functionally graded material, and porous metal foam material. The effects of anisotropy in these composite materials allow designer to efficiently aligning the material's structure with the load paths, therefore, reducing structures' weight without compromising strength. These effects are described through the constitutive relation equations shown below.

Based on Hooke's law, the stress-strain relations for anisotropic materials can be expressed in matrix form as follows:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix}$$

where σ_{ij} are the stress components, ε_{ij} are the axial strain components, and γ_{ij} are the shear strain components.

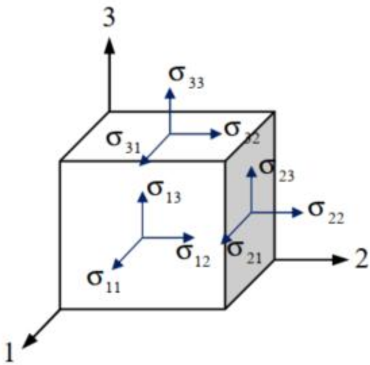

Figure: Stress and strain component in the coordinate system

-Laminated composite material: consist of multiple layers of different fiber-reinforced polymer or metal-matrix composites, stacked in a specific orientation to achieve desired mechanical properties. These materials offer high strength-to-weight ratios, superior stiffness, and customizable anisotropic properties by altering fiber orientation and layer stacking sequences. The behavior of laminated composites is governed by interlaminar stresses, delamination effects, and complex failure mechanisms such as matrix cracking and fiber breakage. Due to their layered nature, higher-order shear deformation theories are often required to capture through-thickness stress variations accurately. Laminated composites are widely used in aerospace, marine, and structural applications where lightweight and high-performance materials are essential. The stiffness components of the LC thin-walled beams L_{ij} $\left(i, j = 1,...,9\right)$ contain the coefficients that can be computed as follows:

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}, B_{sij}, H_{sij}\right) = \sum_{k=1}^{nl}\left(\int_{n_k}^{n_{k+1}}\left(1, n, n^2, f, nf, f^2, ng, g^2\right)Q_{ij}dn\right)$$

where nl is the number of layers and k is the layer index.

-Functionally graded materials (FGM): are advanced materials with a gradual variation in composition and microstructure across their volume, eliminating abrupt interfaces between different material phases. Unlike laminated composites, which have discrete layers, FGMs transition smoothly from one material to another, improving thermal resistance, reducing stress concentrations, and enhancing damage tolerance. These materials are particularly beneficial in high-temperature environments, such as thermal barrier coatings in jet engines and nuclear reactors. The modeling of FGMs often requires shear deformable beam theories to account for the continuous material property variations, making them a key focus in computational mechanics and structural analysis. The effective mass density and are expressed by: $\rho = \rho_c V_c + \rho_m (1 - V_c)$, $E = E_c V_c + E_m (1 - V_c)$ where the subscripts c and m indicate the ceramic and metal, V_c is the volume fraction of ceramic. For gradual change of FG material:

$$V_c = \left[\frac{n}{h_j} + 0.5 \right]^p, \; -0.5h_j \le n \le 0.5h_j \, .$$

For FG sandwich:

$$V_c = \left[\frac{-|n| + 0.5h_j}{0.5(1-\alpha_j)h_j} \right]^p, -0.5h_j \le n \le -0.5\alpha_j h_j$$

$$0.5\alpha_j h_j \le n \le 0.5h_j$$

$$V_c = 1, \; -0.5\alpha_j h_j \le n \le 0.5\alpha_j h_j$$

where p is the power-law index, h_j $(\mathrm{j} = 1,2,3)$ are the thicknesses of the top flange, bottom flange and web; α_j $(j = 1,2,3)$ are the thickness ratio of the ceramic for the top flange, bottom flange and web, respectively.

-Porous metal foam material (PMF): are lightweight, highly porous structures with excellent energy absorption, thermal insulation, and damping properties. These materials are typically produced using controlled foaming processes or additive manufacturing techniques, resulting in a cellular structure with interconnected pores. The mechanical behavior of porous metal foams depends on porosity distribution, pore shape, and material composition. Due to their high compressibility and complex microstructure, advanced constitutive models and homogenization techniques are often used to predict their performance under mechanical and thermal loads.

Symmetric pore distribution (Type A)

$$E(n) = E_{\max}\left(1 - e_0 \cos\left(\frac{\pi n}{h}\right)\right)$$

$$\rho(n) = \rho_{\max}\left(1 - e_m \cos\left(\frac{\pi n}{h}\right)\right)$$

Asymmetric pore distribution (Type B)

$$E(n) = E_{\max}\left[1 - e_0 \cos\left(\frac{\pi n}{2h} + \frac{\pi}{4}\right)\right]$$

$$\rho(n) = \rho_{\max}\left[1 - e_m \cos\left(\frac{\pi n}{2h} + \frac{\pi}{4}\right)\right]$$

where: $E_{\max}$ and $\rho_{\max}$ are the maximum values of Young's modulus and mass density, respectively; e_0 and $e_m = 1 - \sqrt{1 - e_0}$ are the porosity parameters; $\rho(n)$ is mass density of the PMF thin-walled beams

2.3 Ritz-type hybrid series solution

The Ritz method is a widely used energy-based approach for solving complex structural mechanics problems, particularly in beam, plate, and shell theories. By approximating displacement fields using a series of admissible functions, the method converts the governing equations into an equivalent variational form, facilitating efficient numerical solutions. Traditional Ritz solutions commonly employ polynomial or trigonometric shape functions; however, hybrid series formulations incorporating trigonometric and exponential functions are presented in this thesis. Solution is obtained by minimizing the total energy functional of the system using the Lagrange's equation.

BC	Shape functions $\varphi_j(z)$	
	Exponential	Trigonometric
S-S	$(z-L)\left(1-\frac{z}{L}\right)e^{\frac{-jz}{L}}$	$\sin\left(\frac{\pi z}{L}\right)e^{\frac{-jz}{L}}$
C-F	$\left(\frac{z}{L}\right)^2 e^{\frac{-jz}{L}}$	$\sin^2\left(\frac{\pi z}{2L}\right)e^{\frac{-jz}{L}}$
C-C	$\left(\frac{z}{L}\right)^2\left(1-\frac{z}{L}\right)^2 e^{\frac{-jz}{L}}$	$\sin^2\left(\frac{\pi z}{L}\right)e^{\frac{-jz}{L}}$

Table: Shape functions of Ritz type series

2.4 Stochastic analysis

Stochastic analysis is a branch of computing science and mathematics that study the randomness in a system or process. It is widely used in many fields such as finance, physics, biology and engineering. The theory foundation of this study branch trace back to Norbert Wiener's work in the 1940s, famously called the Wiener process, that investigate the one-dimensional Brownian motion. This theory introduced the idea that a stochastic process could be decomposed into a series of orthogonal polynomial functions of random variables, and has been deeply discussed by Szabados [11]. With the advances of the computing systems in the 1990s, Ghanem and Spanos presented the use of polynomial chaos for arbitrary distributions. Andrew and Askey [12] contributed to the polynomial chaos theory by proposing the Askey scheme which organises the orthogonal polynomials and expands the understanding of the stochastic processes. From the 2000s to present, the increasing computational resources and state-of-the-art numerical algorithms have been propelling the use of the polynomial chaos expansion (PCE) across various fields. [13-15]

In this thesis, the stochastic analysis is applied for thin-walled composite beams to study how the random variables of the beam's parameters affect the beam's mechanical responses. These three techniques are depicted briefly below and more detailed in the chapter Four: Monte Carlo Simulation (MCS) , Polynomial Chaos Expansion (PCE), and Artificial Neural Network (ANN).

-**Monte Carlo Simulation (MCS):** the MCS is simply a technique that generate random input samples, and observe how the randomness impacts the outputs. These inputs and outputs can be probabilistic, and are subjected to the physical nature of the variables. Among the 3 techniques, MCS is the simplest in term of mathematical formulation, and can give the most accurate representation of the beams' outcomes.

However, the accuracy can come with an expensive computing cost, especially when the number of random input variables and samples increases. The PCE and ANN are modern techniques that help overcomes the MCS's problem while achieving the same level of accuracy.

-Polynomial Chaos Expansion (PCE): the PCE represents the uncertain parameters in a system as a series of polynomial of random variables multiplied by respective coefficients. These polynomials are chosen to be orthogonal with respect to the probability distribution of the inputs.

$$\hat{u} \simeq \hat{u}_{PCE}\left(\mathbf{x}\right) = \sum_{i=0}^{P-1} c_i He_i\left(\mathbf{q}\right) + \varepsilon \qquad (1\text{-}21)$$

where responses $\hat{u}$ of FG sandwich thin-walled beams $\hat{u}_{PCE}$ are the responses of interest obtained from PCE; $\mathbf{q}$ is a vector of independent random variables in the PCE space mapped to physical random parameters x; He_i are multivariate orthogonal basis functions, which depends on the probability distribution of the random inputs; c_i are coefficients to be determined so that the residual ε is minimized

In this thesis, the residual ε is minimized by the means of least-square regression and Gaussian quadrature method.

-Artificial Neural Network (ANN): ANNs are computational models inspired by the human brain, used extensively in machine learning. As shown in the Fig. below, there are three main types of layers in ANN:

Input layer, Hidden layer, and Output layer. The hidden layers are where most of the

Even though the ANN has been recently applied for the behaviour prediction of thin-walled beams [16-18], there is no research that conduct a thorough comparison between MCS, PCE, and ANN for the stochastic analysis of thin-walled composite beams. This gap is considered in the latter chapter of this thesis.

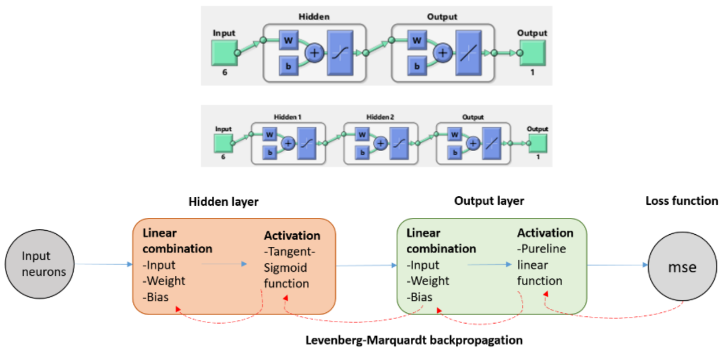

Figure: Artificial neural network workflow

3. Research objectives

The research objectives for this PhD thesis is the study of composite thin-walled beams using MATLAB and higher-order shear deformable theory, coupled with the uncertainty quantification, size-dependent effects on micro-beams, and thermal buckling phenomena, are outlined as follows:

- To develop and validate a computational framework: to implement higher-order shear deformable theory for the analysis of composite thin-walled beams, including laminated, functionally graded, and porous metal foam beams. This framework aims to accurately predict their mechanical behavior under various loading conditions.

- To investigate the impact of material heterogeneity: Study how variations in material composition and distribution within functionally graded, laminated composite, and porous metal foam beams influence their mechanical properties and overall structural performance, using the developed computational model.
- To quantify uncertainties: Employ probabilistic methods to quantify uncertainties related to material properties, geometric imperfections, and loading conditions, and assess their impact on the behavior of composite thin-walled beams.
- To examine the size-dependent effects on micro-beams: Explore the size-dependent behavior of micro-beams made from composite materials, thereby enhancing the understanding of scale effects in micro-scale structures.
- To analyze thermal buckling behavior: Investigate the thermal buckling characteristics of composite thin-walled beams under various thermal loading scenarios. This includes understanding the role of temperature gradients and thermal loads on the stability of these thin-walled beams.

- To validate models through comparative analysis: Compare the predictions of the developed models with existing analytical, numerical, and experimental results from the literature, to validate the accuracy and reliability of the proposed computational framework.

By achieving these objectives, this thesis aims to advance the state of knowledge in the field of composite thin-walled beams, providing a robust computational tool for other researchers, and proposing the optimization approaches for these critical structural elements.

4. Research method

In each of the following chapters, the data are presented in the order based on the research method. The thin-walled beam models and simulations are verified with multiple notable articles and experimental results. Subsequently, the parametric studies are conducted with a range of input parameters fed into this thesis's simulation models. This sequence of research method ensures the accuracy and validity of the new contributions.

5. Conclusion

In conclusion, this thesis has delved into the comprehensive study of thin-walled beams, emphasizing their widespread applications in diverse engineering domains such as civil, aerospace, and automobile engineering due to their notable advantages in loadcarrying capacity and lightweight properties. The investigation primarily focused on understanding the structural responses of thin-walled beams with different cross-section shapes, addressing the static, buckling and dynamic responses to mechanical and thermal loads.

For static analysis, the accurate prediction of beam deflection and buckling stability under varied thermal and mechanical loading conditions is crucial in the design process. Additionally, the thesis explored vibration analysis, scrutinizing the free vibration fundamental frequencies and mode shapes, with a particular emphasis on torsional modes for open-section beams. Building upon established thin-walled beam models, such as Vlasov's model and first-order shear deformable thin-walled beam theory, this thesis proposed a general high-order shear deformable beam theory tailored for thin-walled composite beams. The general high-order shear deformable thin-walled beam theory can better predict the responses of thin-walled beams with low length-to-depth ratio.

The exploration extended to stochastic analysis, where the inherent randomness in constituent material properties was considered. This led to the development of a novel beam solver employing hybrid series-type approximation functions, coupled with the utilization of polynomial chaos expansion (PCE) and artificial neural network (ANN) techniques for efficient evaluations of stochastic responses. The findings, benchmarked against crude Monte

Carlo simulation, provided valuable insights into the impact of material property uncertainties on stochastic responses, serving as potential benchmarks for the scientific and engineering community. The PCE and ANN employed for thin-walled composite beams in this thesis requires only 256 training samples and much less computational time compared to 10^6 samples of MCS benchmark while maintaining the predicted output accuracy.

Furthermore, the thesis carried out the size-dependent effects analysis, replacing classical continuum mechanics with the modified couple stress theory. Numerical results elucidated the influences of material distribution, span-to-height ratio, and material length scale parameters on the bending and vibration behaviors of microbeams under various boundary conditions. The micro-beam exhibits much stiffer behaviours compared to its macro- counterpart.

Prior to conducting these analyses, rigorous validation of the theoretical and numerical models was performed, ensuring the accuracy, efficiency, and computing time of the analysis software code. The culmination of these studies contributes not only to the theoretical advancements in thin-walled beam modeling but also establishes benchmark results for future scientific and engineering research in this domain. The consideration of efficiency and computing time underscores the practical significance of the findings by the PhD candidate for the broader engineering community.

6. Future directions

The phD candidate plans the following directions for the future researches:

- The thermal and hygro analysis of closed-section thin-walled beams with arbitary cross section shapes
- The incorporation of isogeometric method to replace the Ritz-type hybrid series solution in this thesis
- The effects of geometrical nonlinearity on thin-walled beams
- The effects of other loading conditions (moving loads, impact loads) on thin-walled beams
- The Karhunen-Loeve expansion for the stochastic analysis of thin-walled beams
- Artificial neural network's hyperparameter optimisation

References

1. Bui, X.B., P.H. Ngo, and T.K. Nguyen, *A unified third-order shear deformation theory for static analysis of laminated composite beams.* Journal of Technical Education Science, 2019(55): p. 87-93
2. Bui, X.B., T.K. Nguyen, Q.C. Le, and T.T.P. Nguyen. *A novel two-variable model for bending analysis of laminated composite beams*. in *2020 5th International Conference on Green Technology and Sustainable Development (GTSD)*. 2020.

3. Bui, X.-B., A.-C. Nguyen, N.-D. Nguyen, T.-T. Do, and T.-K. Nguyen, *Buckling analysis of laminated composite thin-walled I-beam under mechanical and thermal loads.* Vietnam Journal of Mechanics, 2023. **45**(1): p. 75-90.https://doi.org/10.15625/0866-7136/17956
4. Bui, X.-B. and T.-K. Nguyen, *Deterministic and stochastic flexural behaviors of laminated composite thin-walled I-beams using a sinusoidal higher-order shear deformation theory.* Mechanics Based Design of Structures and Machines, 2023: p. 1-30.10.1080/15397734.2023.2297840
5. Bui, X.-B., T.-K. Nguyen, N.-D. Nguyen, and T.P. Vo, *A general higher-order shear deformation theory for buckling and free vibration analysis of laminated thin-walled composite I-beams.* Composite Structures, 2022. **295**: p. 115775.https://doi.org/10.1016/j.compstruct.2022.115775
6. Bui, X.-B., T.-K. Nguyen, and P.T.T. Nguyen, *Stochastic vibration and buckling analysis of functionally graded sandwich thin-walled beams.* Mechanics Based Design of Structures and Machines, 2023: p. 1-23.https://doi.org/10.1080/15397734.2023.2165101
7. Bui, X.-B., T.-K. Nguyen, T.T.-P. Nguyen, and V.-T. Nguyen. *Stochastic Vibration Responses of Laminated Composite Beams Based on a Quasi-3D Theory*. in *ICSCEA 2021*. 2023. Singapore: Springer Nature Singapore.

8. Bui, X.-B., T.-K. Nguyen, A. Karamanli, and T.P. Vo, *Size-dependent behaviours of functionally graded sandwich thin-walled beams based on the modified couple stress theory.* Aerospace Science and Technology, 2023. **142**: p. 108664.https://doi.org/10.1016/j.ast.2023.108664
9. Bui, X.-B., P.T.T. Nguyen, and T.-K. Nguyen, *Spectral projection and linear regression approaches for stochastic flexural and vibration analysis of laminated composite beams.* Archive of Applied Mechanics, 2024.https://doi.org/10.1007/s00419-024-02565-x
10. Vlasov, V.Z., *Thin-walled elastic beams*. 1961, Published for the National Science Foundation, Washington, D.C., by the Israel Program for Scientific Translations Jerusalem: Jerusalem
11. Szabados, T., *An elementary introduction to the Wiener process and stochastic integrals.* Studia Scientiarum Mathematicarum Hungarica, 2010. **31**
12. Andrews, G.E. and R. Askey. *Classical orthogonal polynomials*. in *Polynômes Orthogonaux et Applications*. 1985. Berlin, Heidelberg: Springer Berlin Heidelberg.
13. Shen, D., H. Wu, B. Xia, and D. Gan, *Polynomial Chaos Expansion for Parametric Problems in Engineering Systems: A Review*. IEEE Systems Journal, 2020. **14**(3): p. 4500-4514.10.1109/JSYST.2019.2957664

14. Hadigol, M. and A. Doostan, *Least squares polynomial chaos expansion: A review of sampling strategies.* Computer Methods in Applied Mechanics and Engineering, 2018. **332**: p. 382-407.https://doi.org/10.1016/j.cma.2017.12.019

15. Lüthen, N., S. Marelli, and B. Sudret, *Sparse Polynomial Chaos Expansions: Literature Survey and Benchmark.* SIAM/ASA Journal on Uncertainty Quantification, 2021. **9**(2): p. 593-649.10.1137/20M1315774

16. Susac, F., E.F. Beznea, and N. Baroiu. *Artificial neural network applied to prediction of buckling behavior of the thin walled box*. in *Advanced Engineering Forum*. 2017. Trans Tech Publ.

17. Jeon, J., J. Kim, J.J. Lee, D. Shin, and Y.Y. Kim, *Development of deep learning-based joint elements for thin-walled beam structures.* Computers & Structures, 2022. **260**: p. 106714

18. Torregrosa, A., A. Gil, P. Quintero, and A. Cremades, *A reduced order model based on artificial neural networks for nonlinear aeroelastic phenomena and application to composite material beams.* Composite Structures, 2022. **295**: p. 115845